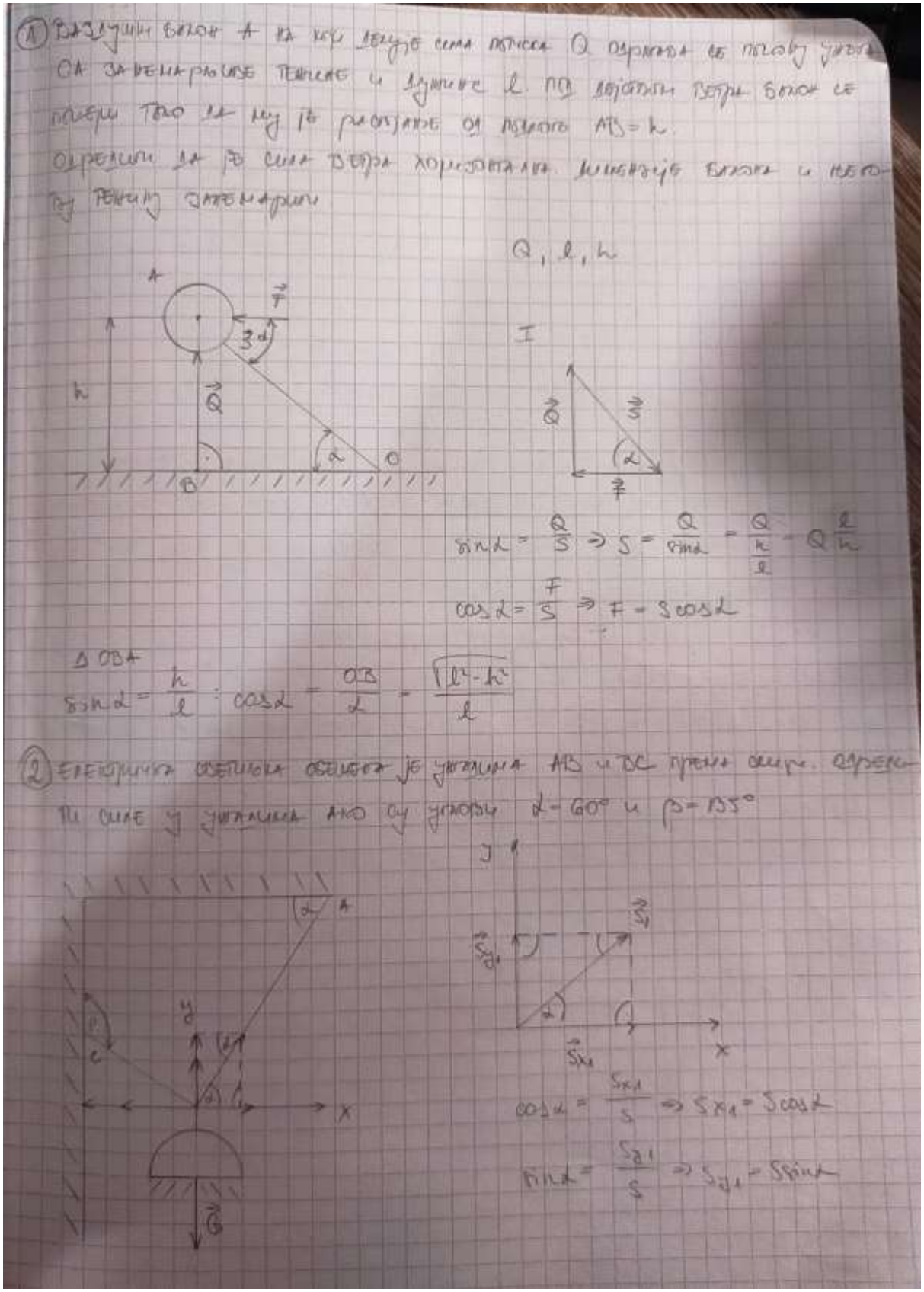
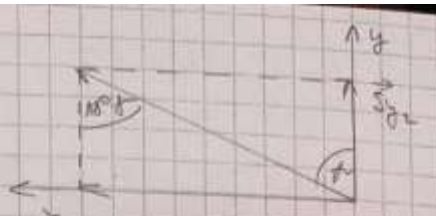
TEHNIČKA MEHANIKA 1 – ZADACI



1) $\sum_{i=1}^{n} F_{xi} = 0 \; ; \; S_{x1} - S_{x2} = 0$

2) $\sum_{i=1}^{n} F_{yi} = 0 \; ; \; S_{y1} + S_{y2} - G = 0$



$\cos\alpha = \frac{S_{y2}}{S} \Rightarrow S_{y2} = S\cos\alpha$

$\sin\alpha = \frac{S_{x2}}{S} \Rightarrow S_{x2} = S\sin\alpha$

$S_1 \cos 60° - S_2 \sin 45° + S_1 \sin 60° + S \cos 45° - G = 0$

$S_1 \cos 60° + S_1 \sin 60° - G = 0$

$S_1 = \frac{G}{\cos 60° + \sin 60°} = 36,6\,N$

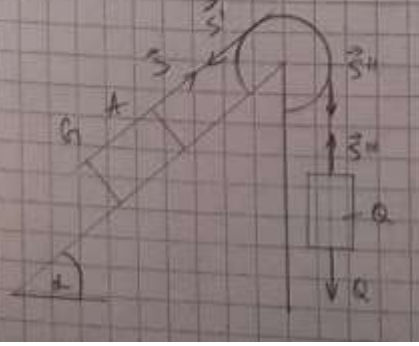
③ Клизач А тежине $G = 10\,kN$ занемарљивих димензија може да клизи по сувој равни под дејством силе теже. За клизач је везано уже које је пребачено преко непокретног котура занемарљивог тежине и које носи тег Q. Права раван је под углом $\alpha = 30°$.

Одредити тежину тега Q за следеће ситуације:

а) када нема трења

б) када је граничка равнотежа у сувоћу покретања клизача на овој равни, а коефицијент трења $\mu = 0,2$

в) када је граничка равнотежа у сувоћу покретања клизача по овој равни, а коеф. трења исто $\mu = 0,1$.



$\alpha = 30°$

$Q = ?$

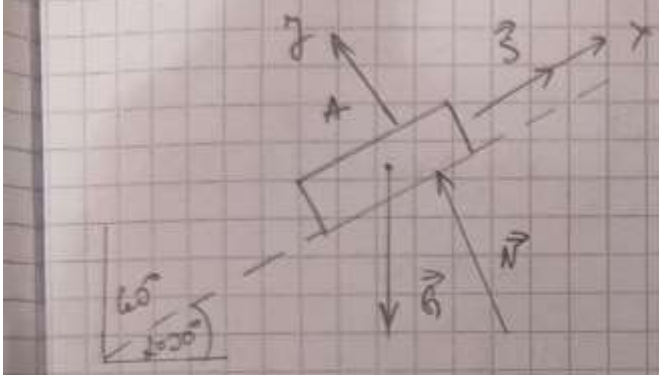
$G = 10$

а) $\mu = 0$

б) $\mu = 0,2$

в) $\mu = 0,2$

a)



$S = S' = S'' = S''' = Q$

1.) $\sum F_{xi} = 0 \qquad S - G \cdot \cos 60° = 0$

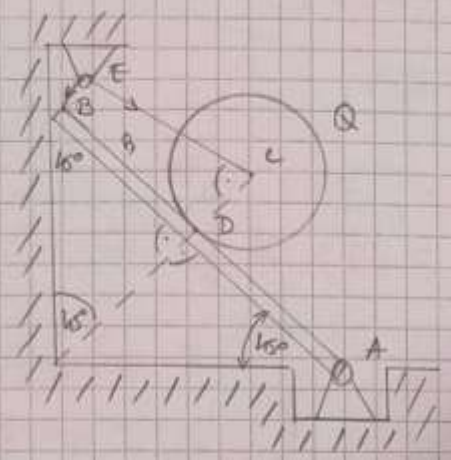
2) $\sum F_{yi} = 0 \qquad N - G \cos 30° = 0$

1.) $Q - G \cos 60° = 0$

$Q - 5\,kN$

2.) $N = G \cos 30° \Rightarrow 5\sqrt{3}\,kN$

⑥



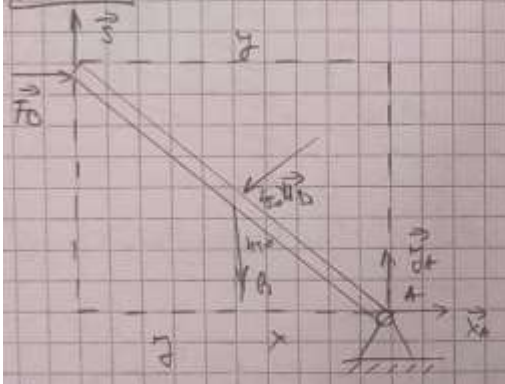
$BD = AD \qquad AB = l$

$G = Q$

$X_A, Y_A, X_B, Y_B = ?$

1. DARAB

2. DARAB



$l^2 = 2y^2 \qquad N_D = S$

$\frac{l^2}{4} = 2x^2 \qquad y = \frac{l\sqrt{2}}{4} \qquad G^2 = 2N_D^2$

$N_D = \frac{G\sqrt{2}}{2}$

$S = \frac{G\sqrt{2}}{2}$

$x^2 = \frac{l^2}{8}$

$x = \frac{l}{2\sqrt{2}} = \frac{l\sqrt{2}}{4}$

1.) $\sum_{i=1}^{n} x_i = 0$:

$X_A - N_D \sin 45° + F_B = 0$

$X_A = N_D \sin 45° - F_B$

$X_A = \frac{G}{2} - G\left(1 - \frac{\sqrt{2}}{2}\right) = G\left(\frac{\sqrt{2}}{2} - \frac{1}{2}\right)$

2.) $\sum_{i=1}^{n} y_i = 0$:

$Y_A - N_D \cos 45° - G + S = 0$

$Y_A = \frac{G\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2} + G - \frac{G\sqrt{2}}{2}$

$Y_A = \frac{2G}{2} - \frac{G\sqrt{2}}{2} =$

$= \frac{G}{2}\left(2 - \sqrt{2}\right)$

3.) $\sum M_A = 0$:

$N_D \frac{l}{2} + Gx - F_{By} - Sy = 0$

$F_{By} = N_D \frac{l}{2} + Gx - Sy$

$F_{By} = \frac{G\sqrt{2}}{2}\cdot\frac{l}{2} + G\frac{l\sqrt{2}}{4} -$

$- \frac{G\sqrt{2}}{2}\cdot\frac{l\sqrt{2}}{2}$

$$F_{By} = G\left(\frac{\sqrt{2}}{4} + \frac{\sqrt{2}}{4} - \frac{1}{2}\right)\ell - G\left(\frac{\sqrt{2}}{2} - \frac{1}{2}\right)\ell$$

$$F_B = G\,\frac{\left(\frac{\sqrt{2}}{2} - \frac{1}{2}\right)\ell}{\ell\,\frac{\sqrt{2}}{2}} = G\left(1 - \frac{1}{\sqrt{2}}\right) = G\left(1 - \frac{\sqrt{2}}{2}\right)$$

(7)

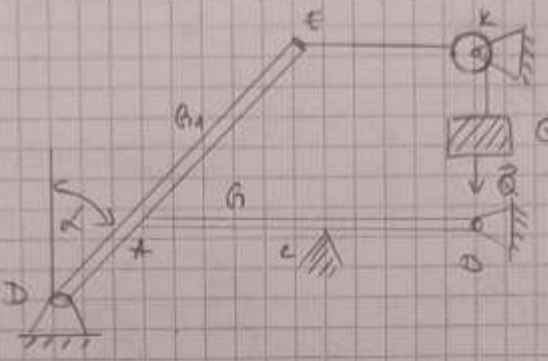
$G = 10\,kN$ $\quad AD = BC = \ell$ $\quad \alpha = 30°$ $\quad \ell = 1\,m$

$AD = 3\ell$

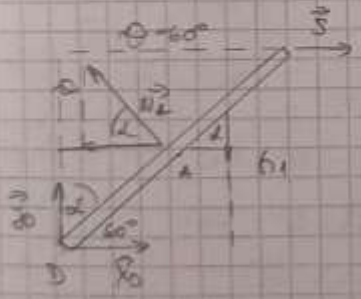
$G_1 = 20\,kN$

$DE = 3\ell$

$Q = 30\,kN$



1. Тело G_1 $\quad S = Q$



1) $\sum_{i=1}^{2} X_i = 0 : \quad X_D - N_A\cos\alpha + S = 0$

2) $\sum_{i=1}^{2} Y_i = 0 : \quad Y_D - G_1 + N_A\sin\alpha = 0$

3) $\sum \overset{\circlearrowleft}{M_D} = 0 : \quad \ell\,N_A - G_1\,\frac{\alpha\ell}{2} - S\,\alpha\ell\cos\alpha = 0$

$\ell N_A = G_1\,\frac{\alpha\ell}{4} + S\,\frac{\alpha\ell}{2}\sqrt{3}$

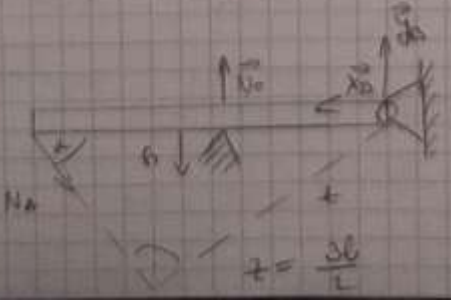
$N_A = \frac{3}{2}\,(10 + 30\sqrt{3}) = 144,3\,k$

$X_D = N_A\cos 30° - Q =$

$= 144,3\,\frac{\sqrt{3}}{2} - 30 = 75,5\,kN$

$Y_D = G_1 - N_A\sin 30° = -52,45\,kN$

2. Тело G_1



1) $\sum_{i=1}^{2} X_i = 0 :$

$N_A\cos\alpha - X_B = 0$

$X_B : N_A\cos 30° =$

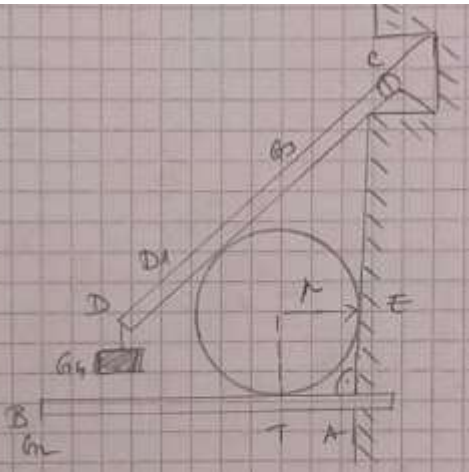
$= 125,49\,kN$

③

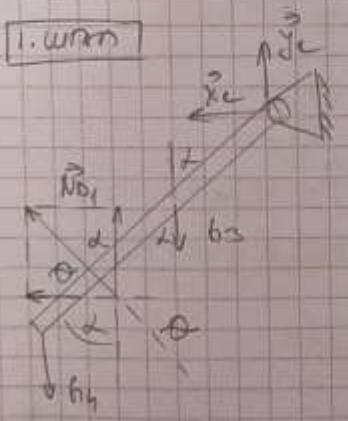
$AC = CD = AB = 4r$

$G_1 = G_4 = 3G$

$G_2 = G_3 = G$



1. wariant



1) $\sum_{i=1}^{n} x_i = 0 : \quad -x_C - N_{D_1}\cos\alpha = 0$

2) $\sum_{i=1}^{n} y_i = 0 : \quad -G_4 - G_3 + N_{D_1}\sin\alpha + y_C = 0$

3) $\sum M_C = 0 :$

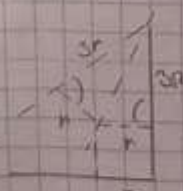
$2r\,G_3\sin\alpha + G_4 \cdot 4r\sin\alpha - N_{D_1} \cdot 3r = 0$

$3N_{D_1} = 2G\sin\alpha + 4G\sin\alpha$

$N_{D_1} = \frac{2G\sin\alpha + 12G\sin\alpha}{3} =$

$= (G\sin 36{,}87^\circ)\,\frac{14}{3} = 2{,}8G$

$x_C = -N_{D_1}\cos 36{,}87^\circ = -2{,}24G$



$\operatorname{tg}\frac{\alpha}{2} = \frac{r}{3r}$

$\frac{\alpha}{2} = \operatorname{arctg}\frac{1}{3}$

$\alpha = 2\operatorname{arctg}\frac{1}{3}$

$\alpha = 36{,}87^\circ$

$y_C = 3G + G - 2{,}8G\sin 36{,}87^\circ$

$y_C = 4G - 1{,}68G$

$y_C = 2{,}32G$

2. Кугла

1) $\sum_{i=1}^{n} X_i^{\circ} = 0$: $N_{D1}\cos\theta - N_E = 0$

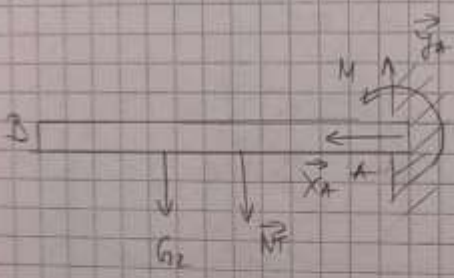
$N_E = N_{D1}\cos 36{,}87 = 2{,}24G$

2) $\sum_{i=1}^{n} Y_i^{\circ} = 0$: $-N_{D1}\sin\theta - G_1 + N_T = 0$

$N_T = 2{,}8G\sin 36{,}87 + 3G$

$N_T = 4{,}68G$

3. Штап 2



1) $-X_A = 0$

2) $Y_A - G_2 - N_T = 0$

$Y_A = G + 4{,}68G = 5{,}68G$

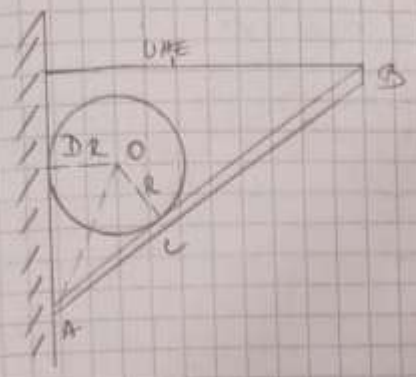
3) $\overset{\curvearrowleft}{M_B} = 0$:

$-2r\,G_2 - 3r\,N_T + 4r\,Y_A + M = 0$

$M = -2rG + 3r\cdot 4{,}68G - 4r\cdot 5{,}68G =$

$= -6{,}68\,rG$

(10)



$AO = 2R$

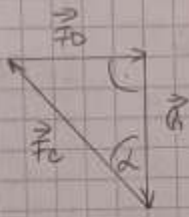
$AB = 6R$

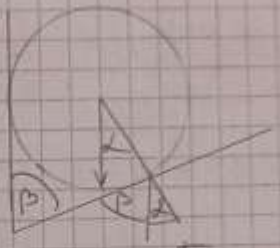
$\sin\frac{\beta}{2} = \frac{R}{2R}$

$\frac{\beta}{2} = 30°$

$\beta = 60°$

1. sposób





$F_D = \frac{F_C}{2}$

$F_C^2 = \frac{F_C^2}{4} + G^2$

$\frac{3}{4}F_C^2 = G^2$

$F_C = \frac{2G\sqrt{3}}{3}$ $F_D = \frac{G\sqrt{3}}{3}$

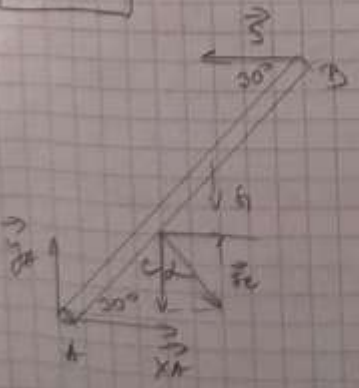
$\alpha = \frac{\pi}{2} - \beta$

$\alpha = 30°$

$AC = \sqrt{4R^2 - R^2}$

$AC = R\sqrt{3}$

2. sposób



1) $\sum_{i=1}^{2} x_i = 0 : X_A + F_C \sin\alpha - S = 0$

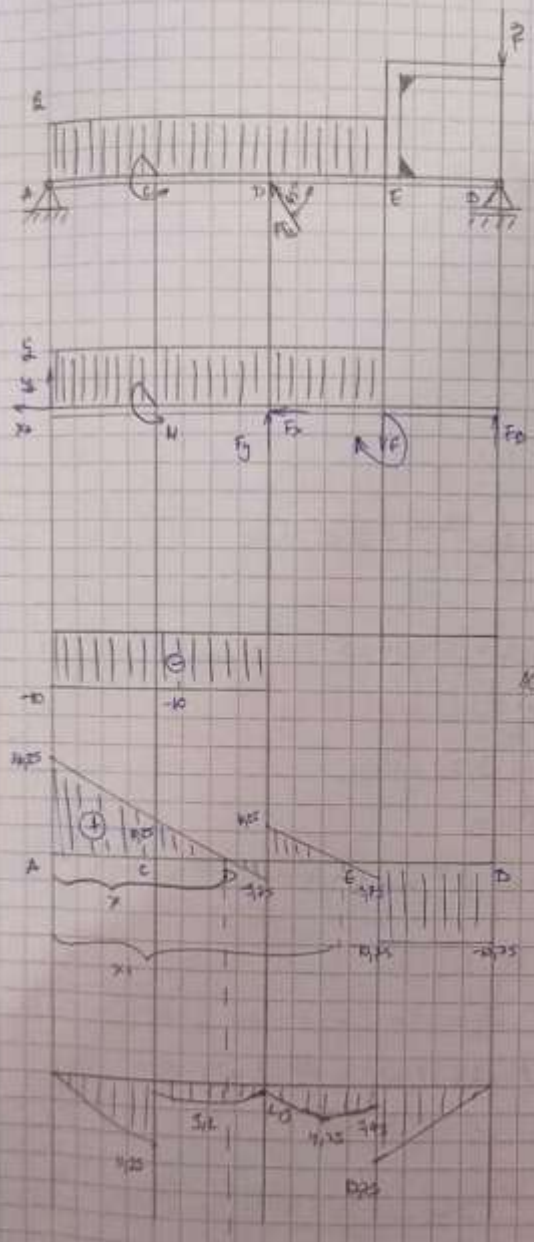
2) $\sum_{i=1}^{2} y_i = 0 : y_A - F_C \cos\alpha - G = 0$

$y_A = \frac{2G\sqrt{3}}{3}\cdot\frac{\sqrt{3}}{2} + G = 2G$

3) $\sum \vec{M}_A = 0 : -F_C \cdot AC - G \cdot 3R \cos 30° + S \sin 30° \cdot 6R = 0$

$S = \frac{G}{3}\left(2 + \frac{3\sqrt{3}}{2}\right)$

(b) Dane: $M = 10\ kNm$, $F = 10\ kN$, $q = 10\ kN/m$, $l = 1\ m$



$M_F = F \cdot l = 10\ kN$

$F_x = F \sin\frac{\pi}{2} = F = 10\ kN$

$F_y = F = 10\ kN$

1) $\sum_{i=1}^{n} X_i = 0$: $-X_A - F_x = 0$ $\quad X_A = -F_x = -10\ kN$

2) $\sum_{i=1}^{n} Y_i = 0$: $Y_A - q \cdot 3l + F_y - F + F_B = 0$

$Y_A - 30 + 10 - 10 + F_B = 0$

$Y_A = 30 - F_B$

3) $\sum M_A = 0$:

$M - q \cdot 3l \cdot \frac{3l}{2} + F_y \cdot 2l - F \cdot 3l - M_F + F_B \cdot 4l = 0$

$10 - 10 \cdot \frac{9}{2} + 10 \cdot 2 - 30 - 10 + F_B \cdot 4 = 0$

$4F_B = 45 + 10$

$F_B = \frac{55}{4} = 13{,}75\ kN$

$Y_A = 30 - 13{,}75 = 16{,}25\ kN$

Sprawdzenie:

$\sum M_B = 0$

$-Y_A \cdot 4l + M + q \cdot 3l \cdot \left(\frac{3l}{2} + l\right) - F_y \cdot 2l + F \cdot l - M_F =$

$= -16{,}25 \cdot 4 + 10 + 30 \cdot \frac{5}{2} - 20 + 10 - 10 = 0$

1) Przedziały linii

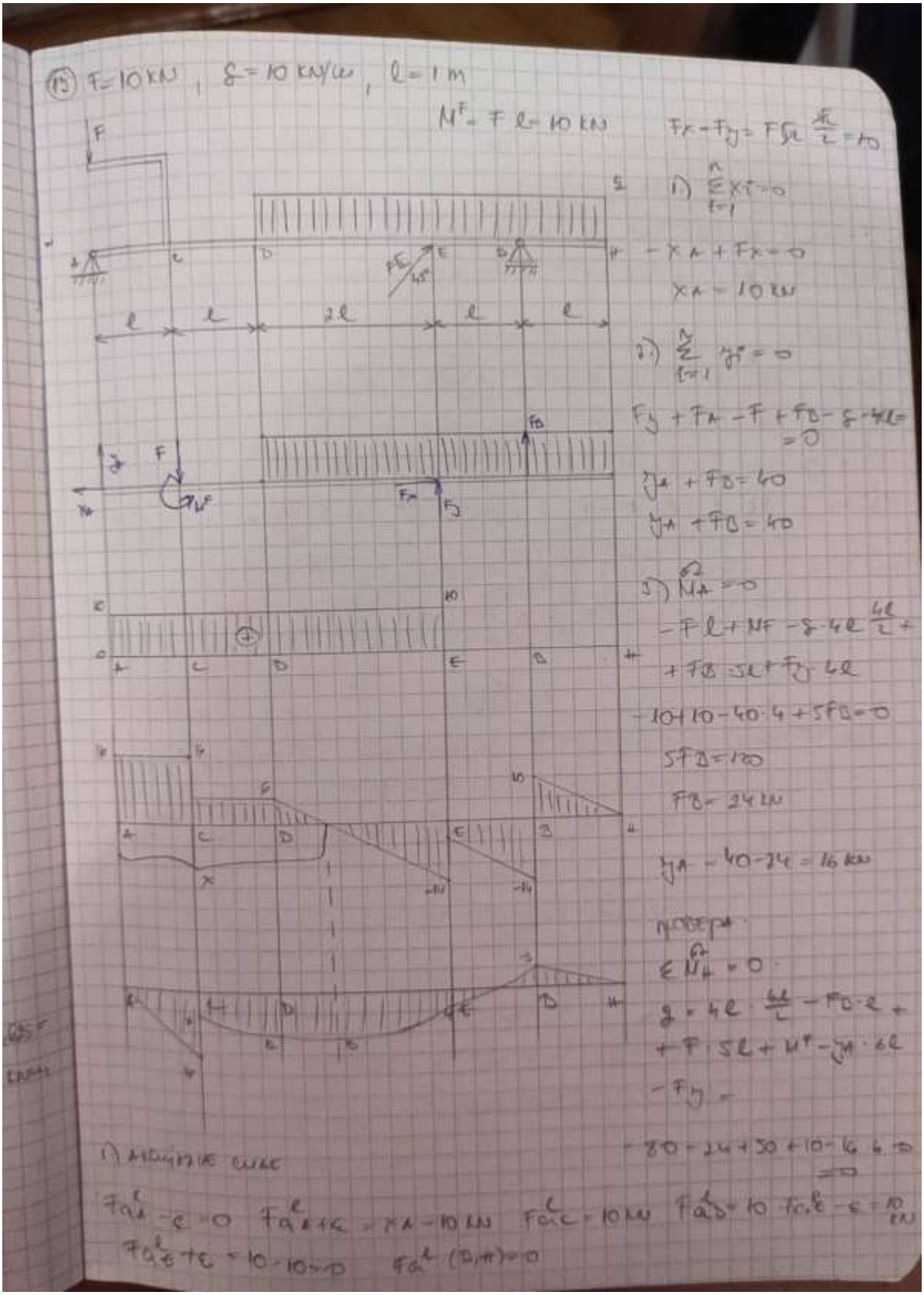
$F_{AA}^{l} \cdot \varepsilon = 0$ $\quad F_{AA}^{L} + \varepsilon = X_A = -10\ kN$

$F_{AC}^{L} = X_A = -10\ kN$ $\quad F_{AB}^{L} - \varepsilon = X_A = -10\ kN$

$F_{AE}^{L} + \varepsilon = X_A + F_x = -10 + 10 = 0$

$F_{AE}^{L} - X_A + F_x = 0$

$F_{AB}^{L} = 0$


(13) F=10 kN, q=10 kN/m, l=1 m
$M^F = F\,l = 10$ kN
$F_x = F_y = F\frac{\sqrt{2}}{2} = 10$
1) $\sum_{i=1}^{n} X_i = 0$
$-X_A + F_x = 0$
$X_A = 10$ kN
2) $\sum_{i=1}^{n} y_i = 0$
$F_y + F_A - F + F_B - q \cdot 4l = 0$
$y_A + F_B = 40$
$y_A + F_B = 40$
3) $\sum M_A = 0$
$-F\,l + M^F - q \cdot 4l \frac{4l}{2} + F_B \cdot 5l + F_y \cdot 6l$
$-10 + 10 - 40 \cdot 4 + 5F_B = 0$
$5F_B = 120$
$F_B = 24$ kN
$y_A = 40 - 24 = 16$ kN
kontrola
$\sum M_A = 0$
$q \cdot 4l \cdot \frac{4l}{2} - F_B \cdot 2l + F \cdot 5l + M^F - y_A \cdot 6l - F_y$
$-80 + 24 + 30 + 10 - 6 \cdot 6 = 0$
1) Analityka belki
$F_{a_A^L} - \varepsilon = 0$ $F_{a_{A+C}^L} = X_A = 10$ kN $F_{a_C^L} = 10$ kN $F_{a_D^L} = 10$ $F_{a_E^L} - \varepsilon = 10$ kN
$F_{a_E^L} + \varepsilon = 10 - 10 = 0$ $F_{a^L}(B,H) = 0$

2) Построение эпюр

$F_{tA}^{l} - \varepsilon = 0$ $\quad F_{tA}^{l} + \varepsilon = y_A = 16\,kN$

$F_{tC}^{l} + \varepsilon = y_A - F = 6\,kN$

$F_{tE}^{l} + \varepsilon = -M + F_B = 10\,kN$

$F_{tA}^{l} = y_A - F + F_y - g \cdot 4\ell + F_B = 16 - 10 + 10 - 40 + 24 = 0$

$F_{tC}^{e} - \varepsilon = y_A = 16\,kN$

$F_{tD}^{l} = 6\,kN$ $\quad F_{tE}^{l} - \varepsilon = 6 - 2 \cdot 2\ell = -$

$F_{tB}^{l} - \varepsilon = y_A - F - g \cdot 3\ell + F_y =$

$= 16 - 10 - 30 + 10 =$

3) Моменты

$M_{SA}^{l} = 0$ $\quad M_{SC}^{e} - \varepsilon = y_A \cdot \ell = 16\,kNm$ $\quad M_{SC}^{e} + \varepsilon = y_A \cdot \ell - M^F = 6\,kNm$

$M_{SD}^{l} = y_A \cdot 2\ell - F \cdot \ell - M^F = 16 \cdot 2 - 10 - 10 = 12\,kNm$

$M_{SE}^{l} = y_A \cdot 4\ell - F \cdot 3\ell - M^F - g \cdot 2\ell \left(\frac{2\ell}{2}\right) = 16 \cdot 4 - 10 \cdot 3 - 10 - 10 \cdot 2 = 64 - 60 = 4\,kN$

$M_{SB}^{l} = y_A \cdot 5\ell - F \cdot 4\ell - M^F + F_y \cdot \ell - g \cdot 3\ell \cdot \frac{3\ell}{2} = 16 \cdot 5 - 40 - 10 + 10 - 10 \frac{9}{2} =$

$= -$

$M_{SA}^{l} = y_A \cdot 6\ell - F \cdot 5\ell - M^F + F_y \cdot 2\ell + F_B \cdot \ell - g \cdot 4\ell \frac{4\ell}{2} =$

$= (6 \cdot 6 - 50 - 10 + 20 + 24 - 10)8 = 0\,kNm$

4) Построение эпюр

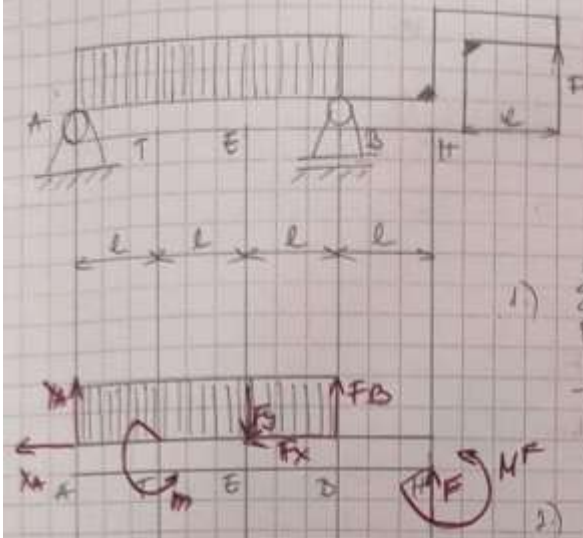
$y_A - F - g \cdot x_1 = 0$

$16 - 10 - 10 \cdot x_1 = 0$

$-10x_1 = -6$ $\quad x_1 = 0{,}6\,m$ $\quad x = 2\ell + x_1 = 4{,}6\,m$

$M_{max} = y_A \cdot x - F \cdot (x - \ell) - M^F - g \cdot x_1 \cdot \frac{x_1}{2} = 16 \cdot 2{,}6 - 10 \cdot 1{,}6 - 10 - \frac{10}{2} 0{,}6^2 =$

15. $M = 10\ kNm$, $F = 10\ kN$, $q = 10\ kN/m$, $l = 1\ m$



$F_x = F\cos\frac{\pi}{4} = 10\ kN$

$F_y = F\sin\frac{\pi}{4} = 10\ kN$

$M^F = F \cdot l = 10\ kN$

1) $\sum_{i=1}^{n} X_i = 0$:

$-X_A - F_x = 0$

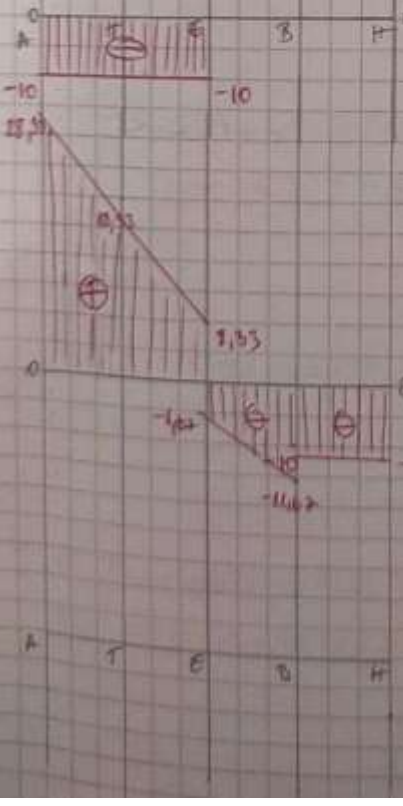
$X_A = -F_x = -10\ kN$

2) $\sum_{i=1}^{n} Y_i = 0$:

$Y_A - F_y + F_B + F - q \cdot 3l = 0$

$Y_A - 10 + F_B + 10 - 30 = 0$

$Y_A + F_B = 30$



3) $\sum M_A^{(F)} = 0$:

$-q \cdot 3l \cdot \frac{3l}{2} + M - F_y \cdot 2l + F_B \cdot 3l + M^F + F \cdot 4l = 0$

$-45 + 10 - 20 + 3F_B + 10 + 10 = 0$

$3F_B = 5$

$F_B = 1{,}67\ kN$

$Y_A = 30 - 1{,}67 = 28{,}33\ kN$

Momenti:

$\sum M_H^{(H)} = 0$:

$M^F - F_A \cdot l + F_y \cdot 2l + q \cdot 3l\left(\frac{3l}{2} + l\right) + M - Y_A \cdot 4l = 0$

$10 - 1{,}67 + 10 + 30 \cdot \frac{7}{2} + 10 - 28{,}33 \cdot 4 = 0$