Trigonometrija

Osnovne trigonometrijske identičnosti

$$\sin^2 x + \cos^2 x = 1 \qquad ctgx = \frac{1}{tgx}$$
$$tgx = \frac{\sin x}{\cos x} \qquad \sin x = \frac{tgx}{\pm\sqrt{1+tg^2x}}$$
$$ctgx = \frac{\cos x}{\sin x} \qquad \cos x = \frac{1}{\pm\sqrt{1+tg^2x}}$$

Svođenje na trigonometrijske funkcije oštrog ugla

$$\sin\left(\tfrac{\pi}{2} \pm x\right) = \cos x \qquad \sin(\pi \pm x) = \mp\sin x \qquad \sin\left(\tfrac{3\pi}{2} \pm x\right) = -\cos x$$
$$\cos\left(\tfrac{\pi}{2} \pm x\right) = \mp\sin x \qquad \cos(\pi \pm x) = -\cos x \qquad \cos\left(\tfrac{3\pi}{2} \pm x\right) = \pm\sin x$$
$$\operatorname{tg}\left(\tfrac{\pi}{2} \pm x\right) = \mp\operatorname{ctg} x \qquad \operatorname{tg}(\pi \pm x) = \pm\operatorname{tg} x \qquad \operatorname{tg}\left(\tfrac{3\pi}{2} \pm x\right) = \mp\operatorname{ctg} x$$
$$\operatorname{ctg}\left(\tfrac{\pi}{2} \pm x\right) = \mp tgx \qquad \operatorname{ctg}(\pi \pm x) = \pm\operatorname{ctg} x \qquad \operatorname{ctg}\left(\tfrac{3\pi}{2} \pm x\right) = \mp\operatorname{tg} x$$

$$\sin(2\pi \pm x) = \pm\sin x \qquad \sin(2k\pi + x) = \sin x \qquad \sin(-x) = -\sin x$$
$$\cos(2\pi \pm x) = \cos x \qquad \cos(2k\pi + x) = \cos x \qquad \cos(-x) = \cos x$$
$$\operatorname{tg}(2\pi \pm x) = \pm\operatorname{tg} x \qquad \operatorname{tg}(k\pi + x) = \operatorname{tg} x \qquad \operatorname{tg}(-x) = -\operatorname{tg} x$$
$$\operatorname{ctg}(2\pi \pm x) = \pm\operatorname{ctg} x \qquad \operatorname{ctg}(k\pi + x) = \operatorname{ctg} x \qquad \operatorname{ctg}(-x) = -\operatorname{ctg} x$$

gde je $k = 0, \pm1, \pm2, \ldots$

Trigonometrijske funkcije zbira i razlike uglova

$$\sin(x \pm y) = \sin x \cos y \pm \sin y \cos x \qquad \operatorname{tg}(x \pm y) = \frac{\operatorname{tg} x \pm \operatorname{tg} y}{1 \mp \operatorname{tg} x \operatorname{tg} y}$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y \qquad \operatorname{ctg}(x \pm y) = \frac{\operatorname{ctg} x \operatorname{ctg} y \mp 1}{\operatorname{ctg} x \pm \operatorname{ctg} y}$$

Trigonometrijske funkcije dvostrukog ugla

$$\sin 2x = 2 \sin x \cos x \qquad tg2x = \frac{2tgx}{1 - tg^2 x}$$

$$\cos 2x = \cos^2 x - \sin^2 x \qquad ctg2x = \frac{ctg^2 x - 1}{2ctgx}$$

Trigonometrijske funkcije polovine ugla

$$\sin \tfrac{x}{2} = \pm \sqrt{\tfrac{1 - \cos x}{2}} \qquad \operatorname{tg} \tfrac{x}{2} = \pm \sqrt{\tfrac{1 - \cos x}{1 + \cos x}}$$

$$\cos \tfrac{x}{2} = \pm \sqrt{\tfrac{1 + \cos x}{2}} \qquad \operatorname{ctg} \tfrac{x}{2} = \pm \sqrt{\tfrac{1 + \cos x}{1 - \cos x}}$$

Transformacija zbira i razlike trigonometrijskih funkcija u proizvod i obrnuto

$$\sin x + \sin y = 2 \sin \tfrac{x+y}{2} \cos \tfrac{x-y}{2} \qquad \sin x \cos y = \tfrac{1}{2}[\sin(x+y) + \sin(x-y)]$$

$$\sin x - \sin y = 2 \cos \tfrac{x+y}{2} \sin \tfrac{x-y}{2} \qquad \sin x \sin y = \tfrac{1}{2}[\cos(x-y) - \cos(x+y)]$$

$$\cos x + \cos y = 2 \cos \tfrac{x+y}{2} \cos \tfrac{x-y}{2} \qquad \cos x \cos y = \tfrac{1}{2}[\cos(x-y) + \cos(x+y)]$$

$$\cos x - \cos y = -2 \sin \tfrac{x+y}{2} \sin \tfrac{x-y}{2}$$

Sinusna teorema: Stranice trougla proporcionalne su sinusima naspramnih uglova, a koeficijent proporcionalnosti je prečnik opisanog kruga oko tog trougla tj.

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma} = 2R.$$

Kosinusna teorema: Kvadrat jedne stranice trougla jednak je zbiru kvadrata ostale dve stranice umanjenog za dvostruki proizvod ovih stranica i kosinusa ugla koji one obrazuju, tj.

$$a^2 = b^2 + c^2 - 2bc \cos \alpha;$$
$$b^2 = a^2 + c^2 - 2ac \cos \beta;$$
$$c^2 = a^2 + b^2 - 2ab \cos \gamma.$$